New Julia and Mandelbrot Sets for Jungck Ishikawa Iterates

Suman Joshi[1], Dr. Yashwant Singh Chauhan[2], Dr. Ashish Negi[3]
[1](Computer Science & Engg. Department,G. B. Pant Engineering College Pauri Garhwal, India)
[2](Computer Science & Engg. Department, G. B. Pant Engineering College Pauri Garhwal, India)
[3](Computer Science & Engg. Department,G. B. Pant Engineering College Pauri Garhwal, India)

ABSTRACT : *The generation of fractals and study of the dynamics of polynomials is one of the emerging and interesting field of research nowadays. We introduce in this paper the dynamics of polynomials $z^n - z + c = 0$ for $n \geq 2$ and applied Jungck Ishikawa Iteration to generate new Relative Superior Mandelbrot sets and Relative Superior Julia sets. In order to solve this function by Jungck –type iterative schemes, we write it in the form of Sz = Tz, where the function T, S are defined as $Tz = z^n + c$ and $Sz = z$. Only mathematical explanations are derived by applying Jungck Ishikawa Iteration for polynomials in the literature but in this paper we have generated Relative Mandelbrot sets and Relative Julia sets.*

Keywords - *Complex dynamics, Relative Superior Mandelbrot set, Relative Julia set, Jungck Ishikawa Iteration*

I. INTRODUCTION

The word "Fractal" which is taken from the Latin word "fractus" meaning "broken" was given by mathematician Benoit B Mandelbrot in 1975[1] to describe irregular and intricate natural phenomenon as lunar landscapes, mountains, branches of trees and coastlines etc. Fractals are defined as "objects that appear to be broken into number of pieces and each piece is a copy of the entire shape". The object Mandelbrot set was given by Mandelbrot in 1979 and its relative object Julia set due to their beauty and complexity of their nature have become rich area of research nowadays.

In Mandelbrot's opinion, the turning point in fractal study occurred in 1970-1980 with his research of the Fatou-julia theory of iteration. This theory had last been changed in 1918. Mandelbrot used a computer to investigate a small portion of Fatou-Julia, which he referred to as the μ-map. It was later renamed the Mandelbrot set (M-set) in his honor by Adrian Douady and John Hubbard.

Fixed point theorem is one of the major tools and it has its diversified applications in the theory of fuzzy mathematics, fractals, theory of games, dynamics programming etc. For a function f having a set X as both domain and range, a fixed point of f is a point x of X for which f(x) =x [2].

II. PRELIMINARIES

1. Ishikawa Iteration [3] Let X be a subset of real or complex numbers and T: X→ X for $x_0 \in X$, we have the sequences $\{x_n\}$ and $\{y_n\}$ in X in the following manner:

$$\left.\begin{aligned} x_{n+1} &= \alpha_n T y_n + (1-\alpha_n) x_n \\ y_n &= \beta_n T x_n + (1-\beta_n) x_n \end{aligned}\right\}$$

where $0 \leq \beta_n \geq 1$ and $0 \leq \alpha_n \geq 1$ and α_n &β_n both convergent to non zero number.

2. Definition [4] The sequences $\{x_n\}$ and $\{y_n\}$ constructed above is called Ishikawa sequences of iteration or relative superior sequences of iterate. We denote it by RSO(x_0, α_n , β_n, t).Notice that RSO (x_0, α_n , β_n, t) with β_n = 1 is RSO(x_0, α_n , t) i.e. Mann's orbit and if we place $\alpha_n = \beta_n = 1$ then RSO (x_0, α_n , β_n, t) reduces to O (x_0, t) .We remark that Ishikawa orbit RSO(x_0, α_n , β_n, t) with β_n = 1/2 is Relative superior orbit. Now we define Julia set for function with respect to Ishikawa iterates. We call them as Relative Superior Julia sets.

3. Definition [4] The set of points SK whose orbits are bounded under Relative superior iteration of function Q (z) is called Relative Superior Julia sets. Relative Superior Julia set of Q is a boundary of Julia set RSK.

4. Jungck Ishikawa Iteration [5]Let $(X, \|.\|)$ be a Banach space and Y an arbitrary set. Let S, T: Y→X be two non self mappings such that $T(Y) \subseteq S(Y)$, S(Y) is a complete subspace of X and S is injective. Then for $x_o \in Y$, define the sequence $\{Sx_n\}$ and $\{Sy_n\}$ iterativelyby

$$\left.\begin{array}{l} Sx_{n+1} = \alpha_n Ty_n + (1-\alpha_n)Sx_n \\ Sy_n = \beta_n Tx_n + (1-\beta_n)Sx_n \end{array}\right\}$$

where n=0, 1 ….. and $0 \le \beta_n \ge 1$ and $0 \le \alpha_n \ge 1$ and α_n & β_n both convergent to non zero number.

III. Figures and Tables

5. Fixed Points

1.1 Fixed points of quadratic polynomial

TABLE 1: Orbit of F (z) for (z_o=-0.3124999945+0.7942708667i) at α =0.5, β =0.5, c=0.1

No. of iterations	|Sz|	No. of iterations	|Sz|
1	0.3124	13	0.1123
2	0.0478	14	0.1124
3	0.0146	15	0.1125
4	0.0546	16	0.1126
5	0.0789	17	0.1126
6	0.0933	18	0.1126
7	0.1016	19	0.1126
8	0.1063	20	0.1126
9	0.1090	21	0.1127
10	0.1106	22	0.1127
11	0.1115	23	0.1127
12	0.1120	24	0.1127

Here we observe that the value converges to a fixed point after 21 iterations.

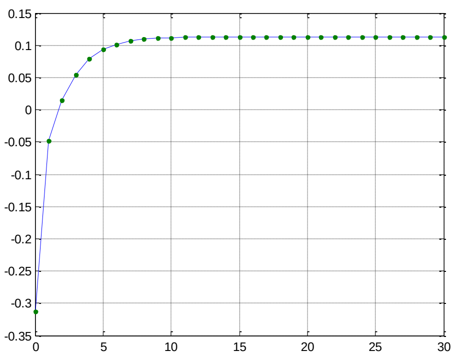


Fig1: Orbit of F(x) for (z_o=0.3124999945+0.7942708667i) at α =0.5, β =0.5, c=0.1

TABLE 2: Orbit of F (z) for (z_0=0.275-1.625i) at α =0.8, β =0.4, c=0.1

No. of iterations	|Sz|	No. of iterations	|Sz|
1	0.2750	11	0.1125
2	0.7462	12	0.1126
3	0.7270	13	0.1126
4	0.4839	14	0.1127
5	0.1549	15	0.1127
6	0.0799	16	0.1127
7	0.0985	17	0.1127
8	0.1078	18	0.1127
9	0.1111	19	0.1127
10	0.1121	20	0.1127

Here we observe that the value converges to a fixed point after 14 iterations .

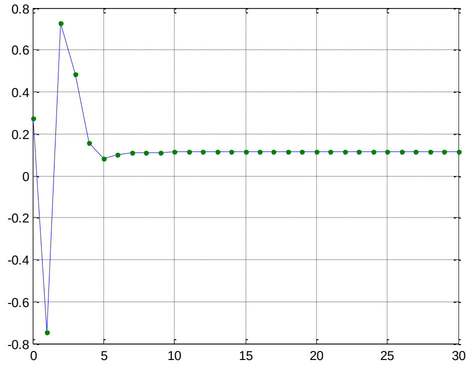


Fig 2. Orbit of F(x) for (x_o=0.275-1.625i) at α =0.8, β =0.4, c=0.1

TABLE 3: Orbit of F (z) for (z_o=1.5-7.6i) at α =0.1, β =0.1, c=0.1

No. of iterations	|Sz|	No. of iterations	|Sz|
140	0.1126	147	0.1127
141	0.1126	148	0.1127
142	0.1126	149	0.1127
143	0.1126	150	0.1127
144	0.1126	151	0.1127
145	0.1126	152	0.1127
146	0.1127	153	0.1127

We skipped 139 iterations and observed that the value converges to a fixed point after 146 iterations.

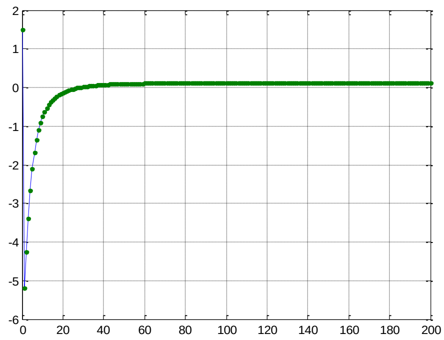


Fig 3: Orbit of F (z) for (z_o=1.5-7.6i) at α =0.1, β =0.1, c=01

1.2 Fixed points of cubic polynomial

TABLE 1: Orbit of F (z) for (z_o=0.09375+0.2625i) at α =0.8, β =0.8, c=0.1

No. of iterations	|Sz|
1	0.0937
2	0.0988
3	0.1005
4	0.1009
5	0.1010
6	0.1010
7	0.1010
8	0.1010
9	0.1010
10	0.1010

Here we observe that the value converges to a fixed point after 6 iterations.

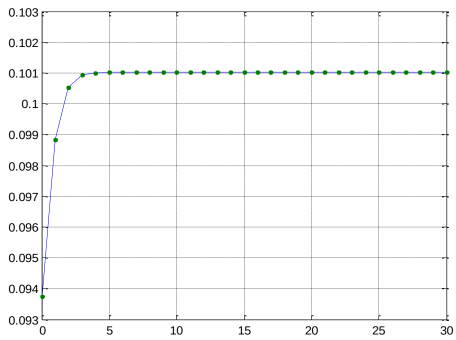


Fig 1 Orbit of F (z) for (z_o=0.09375+0.2625i) at α =0.8, β =0.8, c=0.1

TABLE 2: Orbit of F (z) for (z_o=0.025-1.3875i) at α =0.5, β =0.5, c=0.1

No. of iterations	|Sz|	No. of iterations	|Sz|
1	0.0250	11	0.1008
2	0.0684	12	0.1009
3	0.0838	13	0.1009
4	0.0888	14	0.1010
5	0.0934	15	0.1010
6	0.0967	16	0.1010
7	0.0987	17	0.1010
8	0.0998	18	0.1010
9	0.1004	19	0.1010
10	0.1007	20	0.1010

Here we observe that the value converges to a fixed point after 16 iterations.

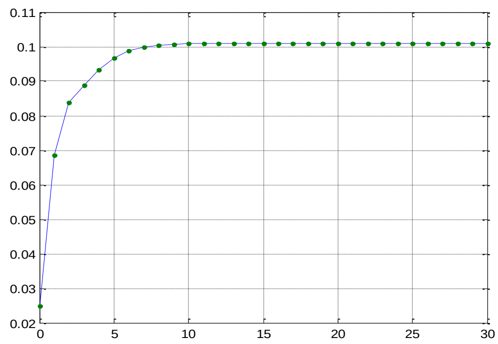


Fig 2: Orbit of F (z) for (z_o=0.025-1.3875i) at α =0.5, β =0.5, c=0.1

TABLE 3: Orbit of F (z) for (z_o=-1.10625-0.39375i)at α =0.4, β =0.6, c=0.1

No. of iterations	|Sz|	No. of iterations	|Sz|
1	1.10625	11	0.10116
2	0.13639	12	0.10111
3	0.12193	13	0.10108
4	0.11184	14	0.10106
5	0.10620	15	0.10105
6	0.10355	16	0.10104
7	0.10232	17	0.10104
8	0.10172	18	0.10103
9	0.10142	19	0.10103
10	0.10125	20	0.10103

Here we observe that the value converges to a fixed point after 18 iterations.

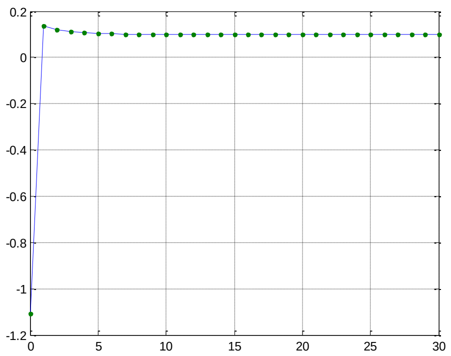


Fig 3: Orbit of F (z) for (z_o=-1.10625-0.39375i) at α =0.4, β =0.6, c=0.1

1.3 Fixed points of biquadratic polynomial

TABLE 1: Orbit of F (z) for (z_o= -0.00625+0.6625i) at α =0.8, β =0.8, c=0.1

No. of iterations	|Sz|
1	0.0062
2	0.0764
3	0.0953
4	0.0991
5	0.0999
6	0.1000
7	0.1000
8	0.1001
9	0.1001
10	0.1001

Here we observe that the value converges to a fixed point after 8 iterations.

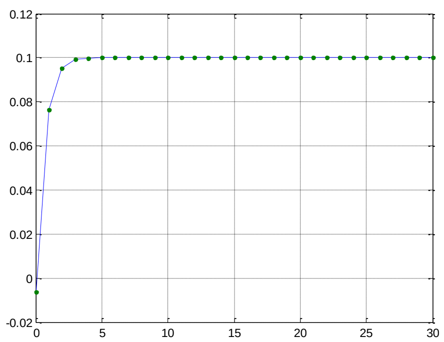


Fig 1: Orbit of F (z) for (z_o= -0.00625+0.6625i) at α =0.8, β =0.8, c=0.1

TABLE 2: Orbit of F (z) for (z_o= -0.06875+1.0875i)at α =0.5, β =0.5, c=0.1

No. of iterations	|Sz|	No. of iterations	|Sz|
1	0.0687	11	0.0988
2	0.4891	12	0.0994
3	0.2088	13	0.0997
4	0.0544	14	0.0999
5	0.0227	15	0.1000
6	0.0613	16	0.1000
7	0.0807	17	0.1000
8	0.0903	18	0.1000
9	0.0952	19	0.1001
10	0.0976	20	0.1001

We observe that the value converges to a fixed point after 19 iterations.

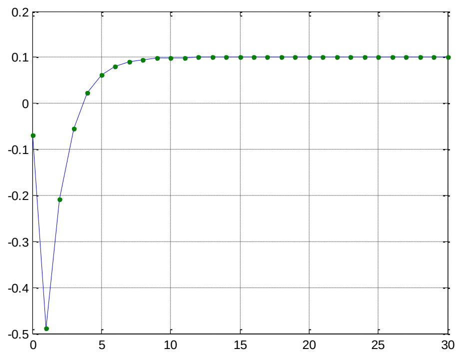


Fig 2: Orbit of F (z) for (z_o= -0.06875+1.0875i) at α =0.5, β =0.5, c=0.1

TABLE 3: Orbit of F (z) for (z_o=-0.26875+1.04375i) at α =0.4, β =0.6, c=0.1

No. of iterations	|Sz|	No. of iterations	|Sz|
1	0.26875	13	0.09991
2	0.04699	14	0.09998
3	0.06819	15	0.10003
4	0.08093	16	0.10006
5	0.08858	17	0.10007
6	0.09318	18	0.10009
7	0.09594	19	0.10009
8	0.09760	20	0.10009
9	0.09860	21	0.10010
10	0.09920	22	0.10010
11	0.09956	23	0.10010
12	0.09978	24	0.10010

Here we observe that the value converges to a fixed point after 21 iterations.

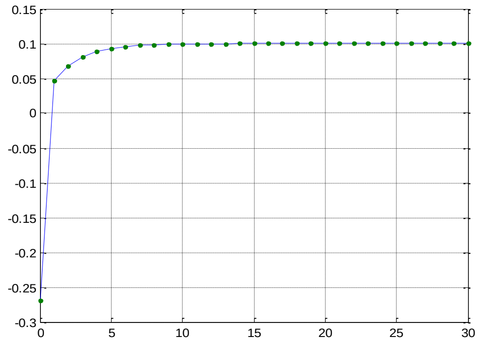


Fig 3: Orbit of F (z) for (z_o=-0.26875+1.04375i) at α =0.4, β =0.6, c=0.1

6. Generation of Relative Superior Julia Sets

We generated the Relative Superior Julia sets. We present here some beautiful filled Relative Superior Julia sets for quadratic, cubic and biquadratic function.

1.4 Relative Superior Julia sets for Quadratic function

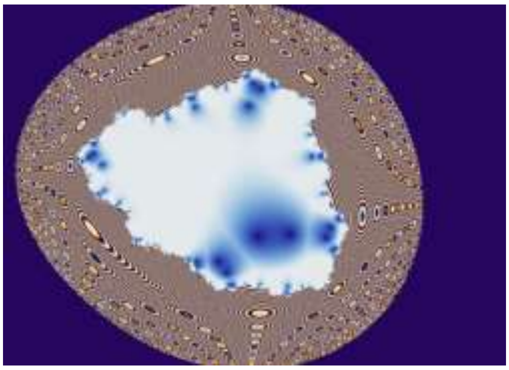


Fig 1: Relative Superior Julia Set for α = β =0.5 & c =0.1

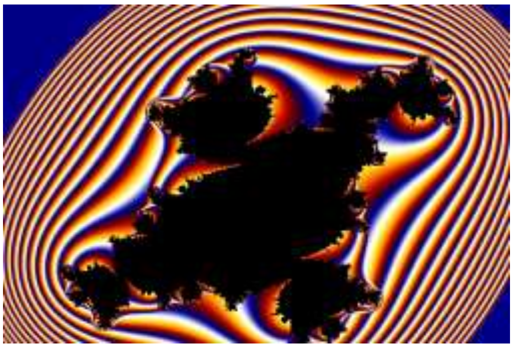


Fig 2: Relative Superior Julia Set for α =0.8, β =0.4, c=0.1

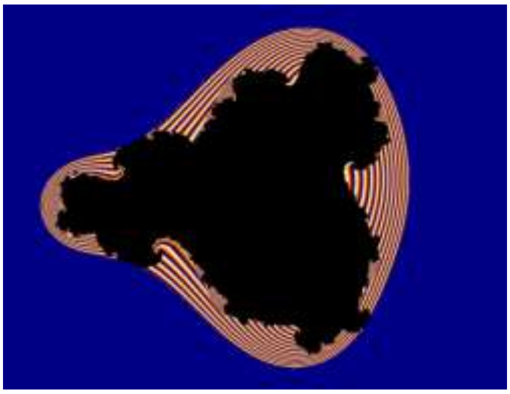


Fig 3: Relative Superior Julia Set for α =0.1, β =0.1, c=0.1

1.5 Relative Superior Julia Sets for Cubic function

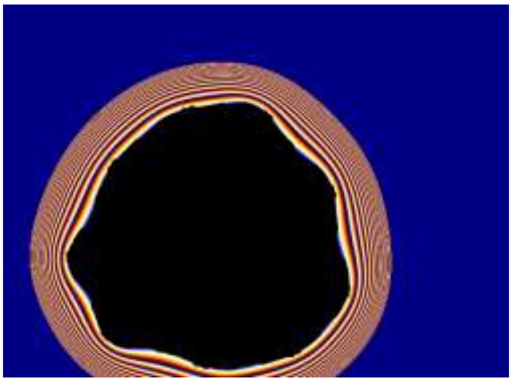


Fig1: Relative Superior Julia Set for $\alpha = \beta$ =0.8, c=0.1

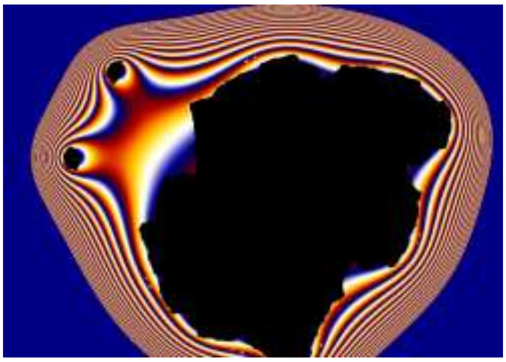


Fig 2: Relative Superior Julia Set for α =0.5, β =0.5, c = 0.1

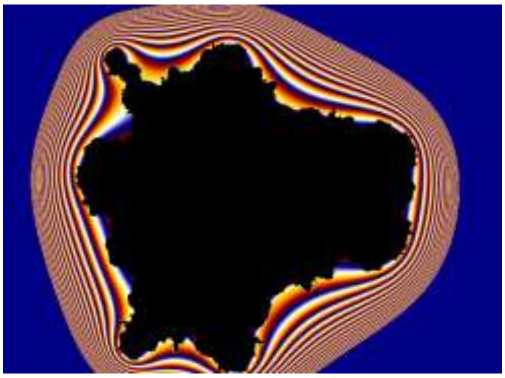


Fig 3: Relative Superior Julia Set for α =0.4, β =0.6, c = 0.1

1.6 Relative Superior Julia sets for biquadratic function

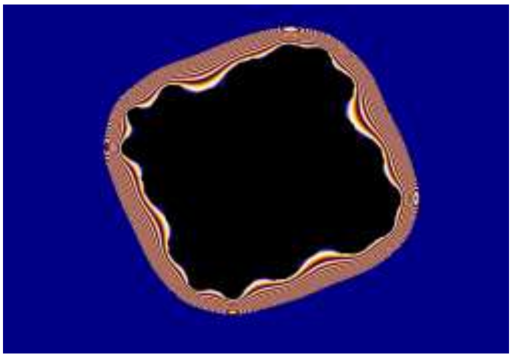


Fig 1: Relative Superior Julia Set for α = 0.8, β = 0.8, c=0.1

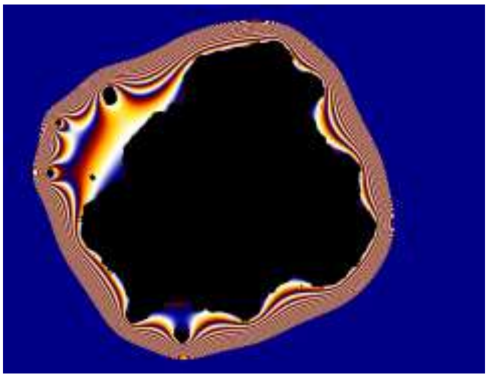


Fig 2: Relative Superior Julia Set for α = 0.5, β = 0.5, c=0.1

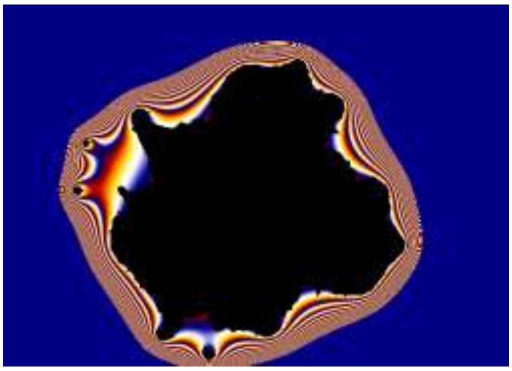


Fig 3: Relative Superior Julia Set for α = 0.4, β = 0.6, c=0.1

7. Generation of Relative Superior Mandelbrot Sets

We generated the Relative Superior Mandelbrot sets. We present here some beautiful filled Relative Superior Mandelbrot sets for quadratic, cubic and biquadratic function.

1.7 Relative Superior Mandelbrot sets for Quadratic function

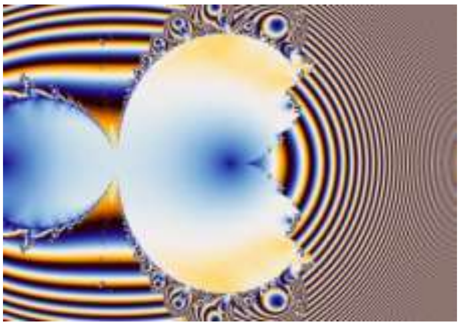


Fig1: Relative Superior Mandelbrot Set for $\alpha = \beta$ =0.5 & c =0.1

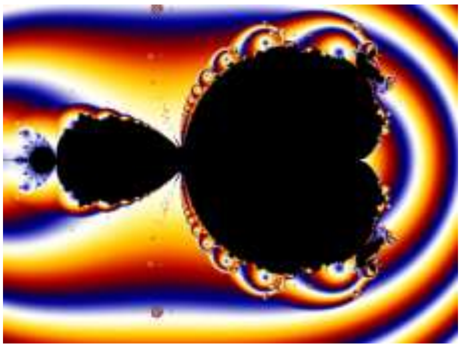
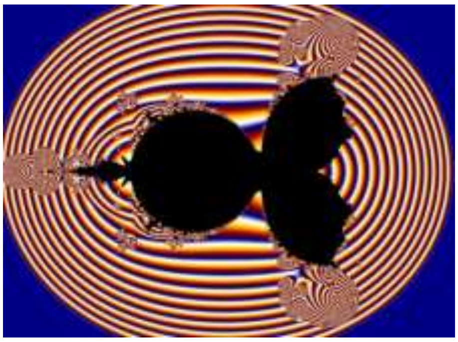


Fig 2: Relative Superior Mandelbrot Set for α =0.8, β =0.4, c=0.1



1.8 Relative Superior Mandelbrot Sets for Cubic function

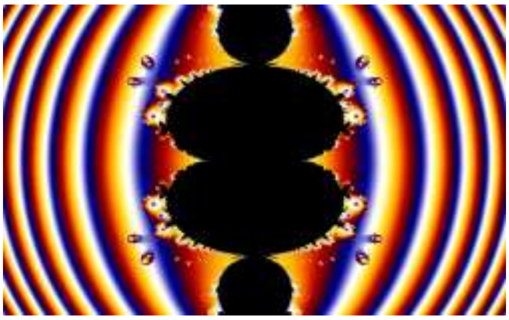


Fig 1: Relative Superior Mandelbrot Set for $\alpha = \beta$ =0.8, c=0.1

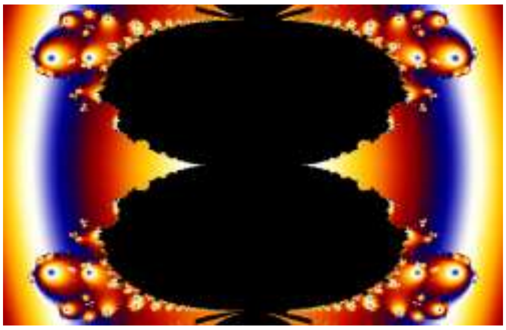


Fig 2: Relative Superior Mandelbrot Set for α =0.5, β =0.5, c = 0.1

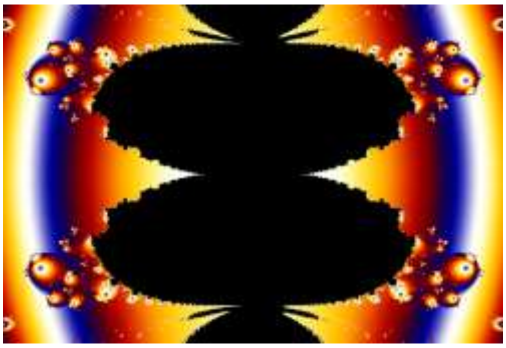


Fig 3: Relative Superior Mandelbrot Set for α =0.4, β =0.6, c = 0.1

1.9 Relative Superior Mandelbrot sets for biquadratic function:

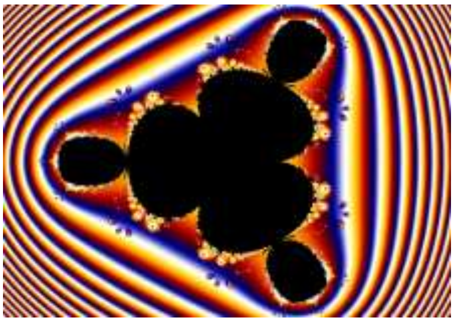


Fig 1: Relative Superior Mandelbrot Set for α =0.8, β =0.8, c = 0.1

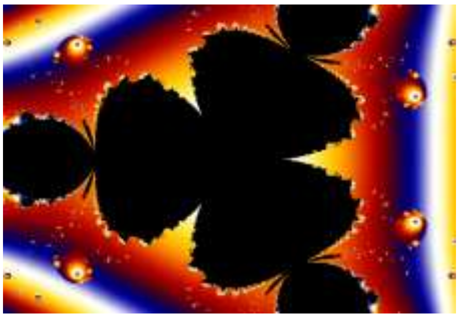


Fig 2: Relative Superior Mandelbrot Set for α =0.5, β =0.5, c = 0.1

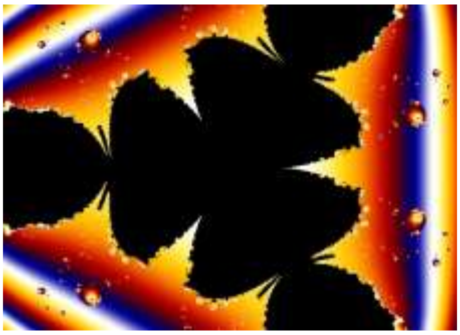


Fig 3: Relative Superior Mandelbrot Set for α =0.4, β =0.6, c = 0.1

IV. CONCLUSION

In the dynamics of complex polynomial $z^n - z + c = 0$ for $n \geq 2$, all the Relative Superior Mandelbrots are symmetrical objects, and for even values of (n) all the Relative Superior Mandelbrots are symmetrical about x-axis and for odd values of (n) all the Relative Superior Mandelbrots are symmetrical about both axis(x-axis and y-axis).

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